Another characterization of tilted algebras

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τ : the AR-translation.

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Definition (Happel-Ringel, 1980's)

An artin A is said to be *tilted* if

$$A = \mathrm{End}_H(T),$$

where H is hereditary and $T \in \mathrm{mod} H$ is tilting.

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2. As an application, we shall show a connection to cluster tilted algebra.

Paths

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A *path* in $\operatorname{ind} A$ is a sequence

$$X_0 \xrightarrow{f_1} X_1 \longrightarrow \cdots \longrightarrow X_{n-1} \xrightarrow{f_n} X_n,$$

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The path is called *non-zero* if $f_1 \cdots f_n \neq 0$.

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(6) *sincere* if every simple A-module is a composition factor of some module in Δ.

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Remark

Δ is slice $\Rightarrow$ Δ is finite, all its components are sections.

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Theorem (Ringel)

An artin algebra A is tilted $\Leftrightarrow$ $\mathrm{mod}A$ has a slice module.

Key Observation

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Then M is a tilting module.

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Remark

A section in Γ_A is a cut, and the converse is not true.

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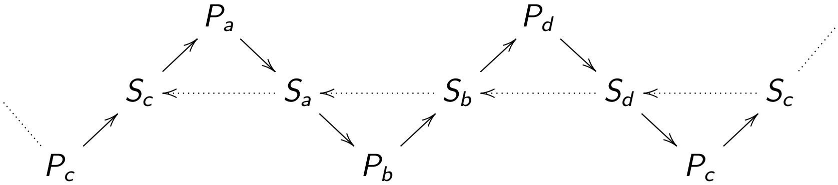

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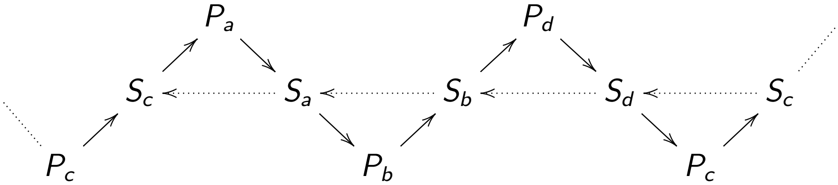

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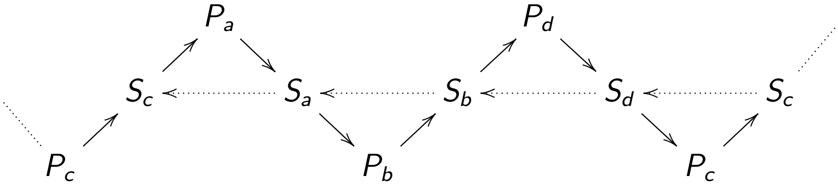

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Δ does not meet the τ-orbits of P_a and P_c.

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2. *$\mathcal{C}$ is preinjective without projective modules.*

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Lemma

If Δ *is* τ*-rigid, then every map* $f : M \to X$, *with* $M \in \Delta$, $X \in \Gamma_A \backslash \Delta$, *factors through some module in* $\tau^{-}\Delta$.

Main Theorem

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REMARK. The faithfulness of Δ cannot be replaced by the sincereness.

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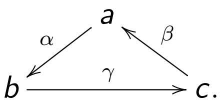

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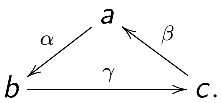

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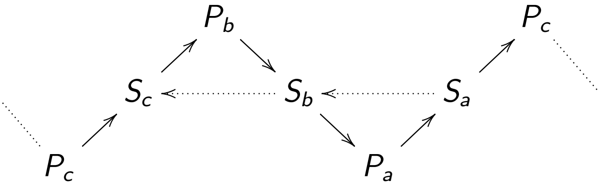

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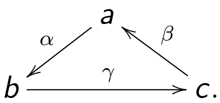

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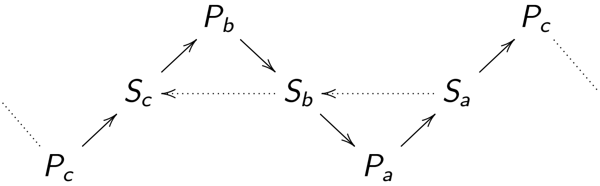

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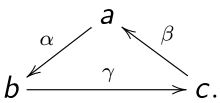

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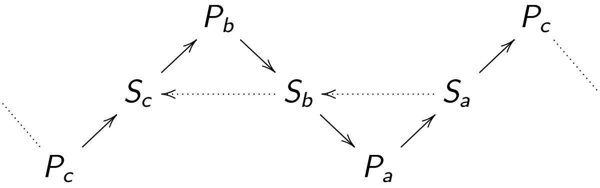

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4. However, A is not tilted.

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Theorem

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2. *B is tilted with Δ being a slice of Γ_B.*

Example

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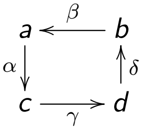

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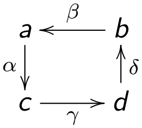

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which is tilted of type $\mathbb{A}_3$.

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Thus, $B = C/\mathrm{ann}(\Delta)$ is tilted with Δ being slice of Γ_B.